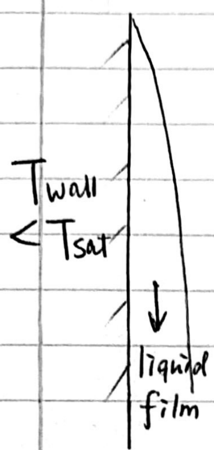
# Condensation Phase Change Heat Transfer

## Film-wise condensation



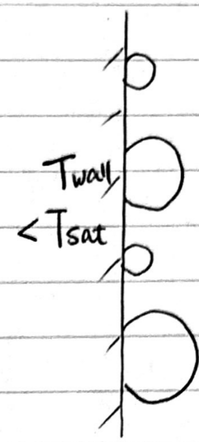

vapor,
$T_{sat}$

$h \sim 10^4 \ W/m^2 \cdot K$

$\downarrow g$

wetting surface

## Dropwise condensation



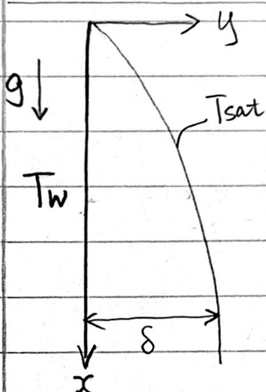

$\downarrow g$

vapor, $T_{sat}$

$h \sim 10^5 \ W/m^2 \cdot K$

non-wetting surface

## Laminar film condensation




saturated vapor, $T_{sat}$

thin-film

$\frac{\delta}{x} \ll 1$

**Momentum**

$$\underbrace{\rho_l \left( u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \right)}_{\text{neglected, small compared to } \mu \frac{\partial^2 u}{\partial y^2}} = -\frac{dP_\infty}{dx} + \mu_l \frac{\partial^2 u}{\partial y^2} + \rho_l g$$

$$\Rightarrow \mu_l \frac{\partial^2 u}{\partial y^2} + g(\rho_l - \rho_v) = 0$$

Boundary conditions: $\begin{cases} (1) & y=0, \quad u=0 \quad \text{no slip} \\ (2) & y=\delta, \quad \tau = 0 \Rightarrow \frac{\partial u}{\partial y} = 0 \end{cases}$

Integrate: $\frac{\partial u}{\partial y} = -\frac{(\rho_l - \rho_v) g}{\mu_l} y + C_1$, $C_1$ obtained from B.C. (2)

Integrate: $u = \frac{(\rho_l - \rho_v) g}{\mu_l} \left( \delta y - \frac{y^2}{2} \right) + C_2$, $C_2 = 0$ from B.C. (1)

$$\Rightarrow \boxed{u(y) = \frac{(\rho_l - \rho_v) g \delta^2}{\mu_l} \left( \frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^2 \right)}$$ velocity distribution.

Define: $\Gamma \equiv$ mass flow rate / width $[kg/ms]$

$$\boxed{\Gamma = \int_0^\delta \rho_l u \, dy = \frac{\rho_l (\rho_l - \rho_v) g \delta^3}{3\mu_l}}$$ mass flow rate

$\boxed{\text{Energy equation}}$

$$\underbrace{u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}}_{\text{neglect, } u,v \text{ small}} = \alpha\frac{\partial^2 T}{\partial y^2}$$

$$\Rightarrow \boxed{\frac{\partial^2 T}{\partial y^2} = 0} \Rightarrow T = C_1 y + C_2$$

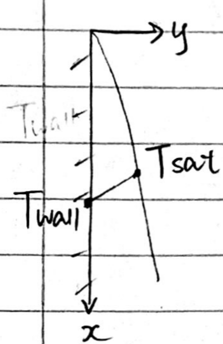
B.C. : (1) $y = 0, \quad T = T_w$

(2) $y = \delta, \quad T = T_{sat}$

$$\Rightarrow \boxed{T - T_w = \frac{y}{\delta}(T_{sat} - T_w)}$$

temperature distribution (linear !)

$\boxed{\text{heat transfer}}$

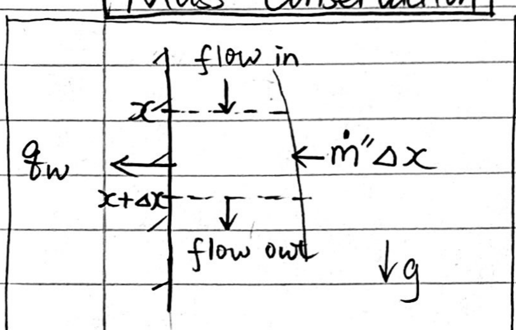



$$q_w = -k_l \frac{\partial T}{\partial y}\Big|_{y=0} = \frac{k_l(T_{sat} - T_w)}{\delta} \Rightarrow \boxed{h = \frac{k_l}{\delta}}$$

Question: what is $\delta$ ?

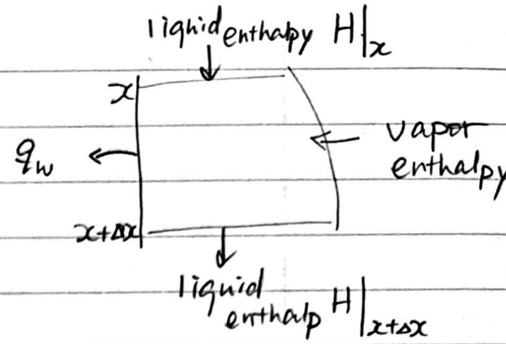
$\boxed{\text{Mass Conservation}}$

define : $\dot{m}''$ = condensation rate [kg/m$^2$.s]




$$\boxed{\dot{m}'' = \frac{d\Gamma}{dx}}$$

Enthalpy flow: (energy balance)




mass of condensed vapor

$$H|_x - H|_{x+\Delta x} + h_v\, d\Gamma - q_w \Delta x = 0$$

specific enthalpy of saturated vapor

specific enthalply of saturated liquid

What is $H$ ? $\quad H = \int_0^{\delta} \rho_l u \left[ h_l - c_{p,l}(T_{sat} - T) \right] dy$

plug in the solution for $T$ : $\quad T - T_w = \frac{y}{\delta}(T_{sat} - T_w)$

$$\Rightarrow H = \left[ h_l - \frac{3}{8} c_{p,l}(T_{sat} - T_w) \right] \Gamma$$

so, $H|_x - H|_{x+\Delta x} = \left[ h_l - \frac{3}{8} c_{p,l}(T_{sat} - T_w) \right] \left( -\Gamma_{x+\Delta x} + \Gamma_x \right)$

$$= \left[ h_l - \frac{3}{8} c_{p,l}(T_{sat} - T_w) \right] (-d\Gamma)$$

so, energy balance becomes :

$$\underbrace{d\Gamma \left[ \underbrace{(h_v - h_l)}_{h_{fg},\ \text{latent heat}} + \frac{3}{8} c_{p,l}(T_{sat} - T_w) \right]}_{} = q_w\, dx$$

$h'_{fg}$ = augmented latent heat of vaporization.

$$\frac{d\Gamma}{dx} = \frac{q_w}{h'_{fg}} = \frac{k_l(T_{sat} - T_w)}{\delta\, h'_{fg}}$$

Note $\frac{d\Gamma}{dx} = \frac{d\Gamma}{d\delta} \cdot \frac{d\delta}{dx} = \frac{\rho_l(\rho_l - \rho_v)g}{3\mu_l} \cdot 3\delta^2 \cdot \frac{d\delta}{dx}$

combining the above 2 equations :

$$\frac{k_l(T_{sat} - T_w)}{\delta\, h'_{fg}} = \frac{\rho_l(\rho_l - \rho_v)g}{3\mu_l} \cdot 3\delta^2 \cdot \frac{d\delta}{dx}$$

$$\Rightarrow dx = \frac{\rho_l(\rho_l - \rho_v)g\, h'_{fg}\, \delta^3}{\mu_l k_l (T_{sat} - T_w)}\, d\delta \Rightarrow x = \frac{\rho_l(\rho_l - \rho_v)g\, h'_{fg}}{4\mu_l k_l (T_{sat} - T_w)}\, \delta^4$$

Finally: $\delta(x) = \left[ \frac{4\mu_l k_l (T_{sat} - T_w) x}{\rho_l(\rho_l - \rho_v) g h'_{fg}} \right]^{1/4}$

$$\Rightarrow h = \frac{k_l}{\delta} = \left[ \frac{k_l^3 \rho_l (\rho_l - \rho_v) g h'_{fg}}{4\mu_l (T_{sat} - T_w) x} \right]^{1/4}$$

$$Nu_x = \frac{hx}{k_l} = \left[ \frac{\rho_l (\rho_l - \rho_v) g h'_{fg} x^3}{4 k_l \mu_l (T_{sat} - T_w)} \right]^{1/4}$$

$$\bar{h} = \frac{1}{L} \int_0^L h \, dx = \frac{4}{3} h \Big|_{x=L}$$

$$\overline{Nu}_L = \frac{\bar{h} L}{k_l} = 0.943 \left[ \frac{\rho_l (\rho_l - \rho_v) g h'_{fg} L^3}{k_l \mu_l (T_{sat} - T_w)} \right]^{1/4}$$

Open question: what if $q_w$ is constant, $T_w$ is unknown?
go through similar procedures.

in energy equation: B.C. ① becomes: $y=0$, $\boxed{k_l \frac{\partial T}{\partial y} = -|q_w|}$

$$\Rightarrow T - T_{sat} = + \frac{|q_w|}{k_l}(y - \delta)$$

(check HW3, P3)

enthalpy becomes: $H = \int_0^\delta \rho_l u \left[ h_l - c_{p,l} \left( \frac{|q_w|}{k_l} (\delta - y) \right) \right] dy$