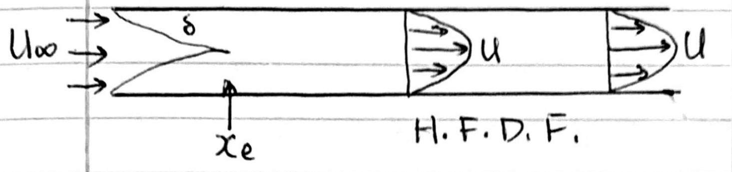
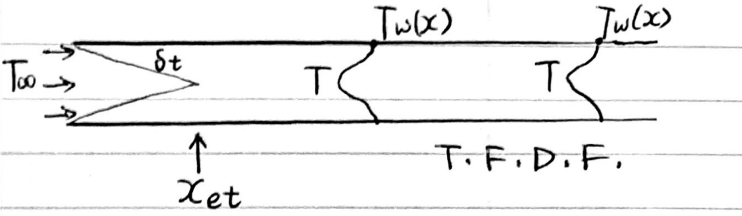
Lecture 6.

Laminar Internal Flows.

Discussions and examples.

Fully developed Flow

Hydrodynamically fully developed (H.F.D.F.)	Thermally fully developed flow (T.F.D.F.)
	
x_e : entry length $\frac{x_e}{D} = 0.03 Re_D$, $Re_D = \frac{U_b D}{\nu}$	x_{et} : thermal entry length. $\frac{x_{et}}{D} \simeq 0.03 Re_D \cdot P_r$
In H.F.D.F. U doesn't change with x. $\boxed{\frac{\partial u}{\partial x} = 0}$ $u(r)$	In T.F.D.F. relative shape of temperature doesn't change with x. $\boxed{\frac{\partial}{\partial x}\left(\frac{T_w - T}{T_w - T_b}\right) = 0}$ $T(x, r)$
governing equation: $\nabla_\perp^2 u = \frac{1}{\mu}\frac{dP}{dx}$ cylindrical coordinate $\frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}u\right) = \frac{1}{\mu}\frac{dP}{dx}$	governing equation: $\underbrace{\alpha \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)}_{\text{heat conduction from wall into fluid}} = \underbrace{u\frac{\partial T}{\partial x}}_{\text{increase in T along pipe}}$
Solution: $u(r) = 2U_b\left(1 - \frac{r^2}{R^2}\right)$ U_b : average velocity $U_b = \frac{R^2}{8\mu}\left(-\frac{dP}{dx}\right)$	solution for constant q_w : $\hookrightarrow \frac{\partial T}{\partial x} = \frac{\partial T_w}{\partial x} = \frac{\partial T_b}{\partial x}$ =constan $T(x,r) = \underbrace{T_w(x)}_{\downarrow \text{ still unknown?}} + \frac{q_w D}{2k}\left\{\left(\frac{r}{R}\right)^2 - \frac{1}{4}\left(\frac{r}{R}\right)^4 - \frac{3}{4}\right.$

Bulk velocity & bulk temperature

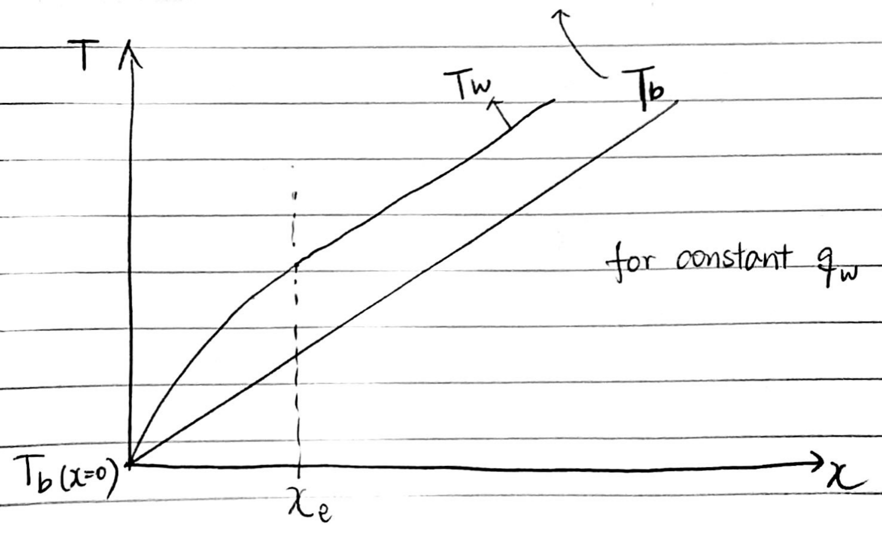
fluid	thermal
bulk velocity is average u.	bulk temperature
$U_b = \frac{1}{A_c}\int_{A_c} u \, dA = \frac{1}{\pi R^2}\int_0^R u \cdot 2\pi r \, dr$	$\boxed{\text{enthalpy} = \dot{m} c_p T_b}$
purpose :	energy balance :
mass flow rate $\dot{m} = \rho U_b A_c$	$\boxed{\underbrace{\frac{\partial(\dot{m} c_p T_b)}{\partial x}}_{\text{fluid energy gain per unit length along tube}} = \underbrace{q_w P}_{\text{energy input from wall.}}}$

Important!

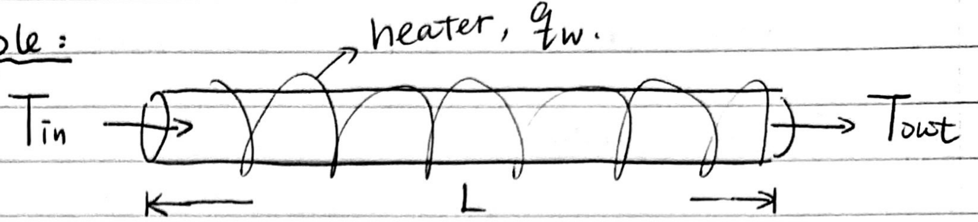
$$\Rightarrow \boxed{\frac{\partial T_b}{\partial x} = \frac{q_w \overset{P = \pi D}{P}}{\underset{\dot{m} = \rho U_b A_c}{\dot{m}} c_p}}$$

Uniform heat flux q_w : T_b change linearly with x.

$$T_b(x) = T_b(x=0) + \frac{q_w P}{\dot{m} c_p} x$$



Example:



water in at 20°C, heater generates $q_w = 2 \times 10^4 \, W/m^2$
tube diameter = 5mm, water average velocity is 0.01 m/s
if we want $T_{out} = 90°C$, how long the tube need to be?

comment: thermophysical properties are typically evaluated at average temperature $\frac{1}{2}(T_{in} + T_{out})$

so, @ $\frac{1}{2}(20+90) = 55°C$,

$\rho = 985 \ kg/m^3$
$C_p = 4184 \ J/kg \cdot K.$

solution:

$$T_b(x=L) = T_b(x=0) + \frac{q_w P}{\rho U_b A_c C_p} \times L$$

$$\Rightarrow L = \frac{T_b(x=L) - T_b(x=0)}{\frac{q_w \cdot \pi D}{\rho U_b \cdot \frac{\pi D^2}{4} \cdot C_p}}$$

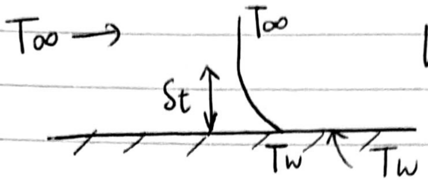
$$= \frac{(90-20)}{\frac{2\times10^4}{985\times0.01\times0.005\times\frac{1}{4}\times4184}}$$

$= 0.18 \ m$

$= 18 \ cm.$

$$\boxed{\text{heat transfer coefficient \& Nusselt number}}$$

external flow



$$h = \frac{q}{T_w - T_\infty}$$

internal flow, thermally F.D.

$$h = \frac{q}{T_w - T_b}$$

① For constant wall heat flux q_w

$$T(x,r) = T_w(x) + \frac{q_w D}{2k}\left\{\left(\frac{r}{R}\right)^2 - \frac{1}{4}\left(\frac{r}{R}\right)^4 - \frac{3}{4}\right\}$$

$$\boxed{T_b} = \frac{\text{enthalpy}}{\dot m C_p} = \frac{\int_0^R \rho U(r) C_p T(r,x)\cdot 2\pi r\,dr}{\int_0^R \rho U(r) C_p \cdot 2\pi r\,dr} = \boxed{T_w - \frac{11}{48}\frac{q_w D}{k}}$$

important: $\boxed{T_b = T_w - \frac{11}{48}\frac{q_w D}{k}}$

comment: q_w positive for heating

q_w negative for cooling.

$$\Rightarrow \boxed{h = \frac{q_w}{T_w - T_b} = \frac{q_w}{\frac{11}{48}\frac{q_w D}{k}} = \frac{48}{11}\frac{k}{D}}$$

$$\boxed{Nu_D = \frac{hD}{k} = \frac{48}{11} = 4.364}$$

② For constant ~~heat flux~~ Wall temperature T_w.

$$\boxed{\begin{array}{l} Nu_D = 3.658 \quad (\text{lower}) \\ h = 3.658\,\frac{k}{D} \end{array}}$$

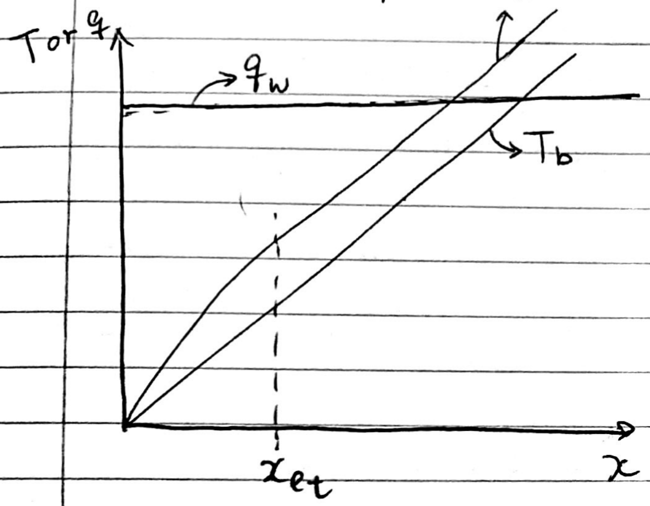
$$\frac{dT_b}{dx} = \frac{q_w P}{\dot m C_p} = \frac{h(x) P}{\dot m C_p}\underbrace{(T_w - T_b)}_{\uparrow\ \Delta T}$$

$$\Downarrow$$

$$\boxed{T_b = T_w + (T_{b,in} - T_w)\,e^{-\frac{hP}{\dot m c_p}x}} \Leftarrow \underbrace{\frac{d\Delta T}{dx} = -\frac{h(x)P}{\dot m C_p}\Delta T}_{\text{exponential!}}$$

T & q plot

constant heat flux



constant wall temperature